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MATHEMATICS
HIGHER LEVEL
PAPER 3 – STATISTICS AND PROBABILITY

Thursday 8 November 2012 (morning)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[60 marks]*.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 16]*

Anna has a box with 10 biscuits in it. 4 biscuits are chocolate and 6 are plain. Anna takes a biscuit from her box at random and eats it. She repeats this process until she has eaten 5 biscuits in total.

Let A be the number of chocolate biscuits that Anna eats.

(a) State the distribution of A. *[1 mark]*

(b) Find $\mathrm{P}(A=3)$. *[2 marks]*

(c) Find $\mathrm{P}(A=5)$. *[1 mark]*

Bill also has a box with 10 biscuits in it. 4 biscuits are chocolate and 6 are plain. Bill takes a biscuit from his box at random, looks at it and replaces it in the box. He repeats this process until he has looked at 5 biscuits in total. Let B be the number of chocolate biscuits that Bill takes and looks at.

(d) State the distribution of B. *[1 mark]*

(e) Find $\mathrm{P}(B=3)$. *[2 marks]*

(f) Find $\mathrm{P}(B=5)$. *[2 marks]*

Let $D=B-A$.

(g) Calculate $\mathrm{E}(D)$. *[2 marks]*

(h) Calculate $\mathrm{Var}(D)$, justifying the validity of your method. *[5 marks]*

2. *[Maximum mark: 11]*

The n independent random variables $X_1, X_2, \ldots, X_n$ all have the distribution $\mathrm{N}(\mu, \sigma^2)$.

(a) Find the mean and the variance of

(i) $X_1 + X_2$;

(ii) $3X_1$;

(iii) $X_1 + X_2 - X_3$;

(iv) $\overline{X} = \dfrac{(X_1 + X_2 + \ldots + X_n)}{n}$. *[8 marks]*

(b) Find $\mathrm{E}(X_1^2)$ in terms of μ and σ. *[3 marks]*

3. *[Maximum mark: 19]*

(a) The random variable X represents the height of a wave on a particular surf beach. It is known that X is normally distributed with unknown mean μ (metres) and known variance $\sigma^2 = \frac{1}{4}$ (metres^2). Sally wishes to test the claim made in a surf guide that $\mu = 3$ against the alternative that $\mu < 3$. She measures the heights of 36 waves and calculates their sample mean $\bar{x}$. She uses this value to test the claim at the 5 % level.

(i) Find a simple inequality, of the form $\bar{x} < A$, where A is a number to be determined to 4 significant figures, so that Sally will reject the null hypothesis, that $\mu = 3$, if and only if this inequality is satisfied.

(ii) Define a Type I error.

(iii) Define a Type II error.

(iv) Write down the probability that Sally makes a Type I error.

(v) The true value of μ is 2.75. Calculate the probability that Sally makes a Type II error. *[11 marks]*

(b) The random variable Y represents the height of a wave on another surf beach. It is known that Y is normally distributed with unknown mean μ (metres) and unknown variance σ^2 (metres^2). David wishes to test the claim made in a surf guide that $\mu = 3$ against the alternative that $\mu < 3$. He is also going to perform this test at the 5 % level. He measures the heights of 36 waves and finds that the sample mean, $\bar{y} = 2.860$ and the unbiased estimate of the population variance, $s_{n-1}^2 = 0.25$.

(i) State the name of the test that David should perform.

(ii) State the conclusion of David's test, justifying your answer by giving the p-value.

(iii) Using David's results, calculate the 90 % confidence interval for μ, giving your answers to 4 significant figures. *[8 marks]*

4. *[Maximum mark: 14]*

Jenny and her Dad frequently play a board game. Before she can start Jenny has to throw a "six" on an ordinary six-sided dice. Let the random variable X denote the number of times Jenny has to throw the dice in total until she obtains her first "six".

(a) If the dice is fair, write down the distribution of X, including the value of any parameter(s). *[1 mark]*

(b) Write down $\text{E}(X)$ for the distribution in part (a). *[1 mark]*

Jenny has played the game with her Dad 216 times and the table below gives the recorded values of X.

Value of X	1	2	3	4	5	6	7	8	9	10	≥ 11
Frequency	40	34	26	24	16	14	12	10	6	4	30

(c) Use this data to test, at the 10 % significance level, the claim that the probability that the dice lands with a "six" uppermost is $\frac{1}{6}$. Justify your conclusion. *[8 marks]*

Before Jenny's Dad can start, he has to throw two "sixes" using a fair, ordinary six-sided dice. Let the random variable Y denote the total number of times Jenny's Dad has to throw the dice until he obtains his second "six".

(d) Write down the distribution of Y, including the value of any parameter(s). *[1 mark]*

(e) Find the value of y such that $\text{P}(Y = y) = \frac{1}{36}$. *[1 mark]*

(f) Find $\text{P}(Y \leq 6)$. *[2 marks]*